Artinian and Noetherian Rings

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The ascending chain condition (ACC) was first introduced by Emmy Noether and then the decending chain condition (DCC) was put forward by Emil Artin. These are algebraic finiteness conditions. Rings that satisfy the ACC and respectively DCC condition on ideals are called Noetherian and respectively Artinian. Noetherian rings satisfy the maximum condition on ideals while Artinian rings satisfy the minimum condition, and these conditions coincide in some cases. In these lectures, we will first study the maximum and minimum conditions on ideals, prove fundamental theorems about them and provide examples. Then the relationship between Artinian and Noetherian rings and questions like whether these properties inherited by subrings and quotients will be investigated. Necessary and sufficient conditions for LPAs to be Noetherian (or Artinian) will be given in terms of combinatorial properties of a digraph Γ and these will be explained with examples.