Parallelisms of $PG(3,4)$ invariant under noncyclic automorphism groups of order 4

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$PG(n,q)$ denotes the n-dimensional projective space over $GF(q)$. A spread in $PG(n,q)$ is a set of lines such that each point is in exactly one line. A parallelism is a partition of the lines of the projective space to spreads [3,4]. Spreads and parallelisms have interesting relations and multiple applications. General constructions are known for $PG(n,2)$, $PG(2^n - 1, q)$ and $PG(3,q)$. All parallelisms of $PG(3,2)$ and $PG(3,3)$ are known [1]. Their classification in larger projective spaces is presently infeasible and only smaller classes are usually concerned.

$PG(3,4)$ is the smallest projective space in which parallelisms have not yet been classified. The parallelisms with nontrivial automorphisms of odd prime orders have already been constructed [5]. There exist, however, plenty of parallelisms with automorphisms of order 2 and their classification is a challenging problem. Parallelisms with cyclic automorphism groups of order 4 have been constructed too [2]. In the present paper we classify the parallelisms invariant under noncyclic groups of order 4. As a result, all the parallelisms of $PG(3,4)$ which possess automorphism groups of order greater than 2 are already known. The problem of the classification of parallelisms with full automorphism groups of order at most 2 remains open.

Keywords
Projective space, parallelism, automorphism, combinatorial design.

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