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Towards the classification of unitals on 28 points of low rank

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Unitals are combinatorial 2-$(q^3+1, q+1, 1)$ designs. The classical examples are the *Hermitian unital* $H(q)$, defined by the absolute points and absolute lines of a unitary polarity in the desarguesian plane of order q^2 and for $q = 3^{2m+1}$ the *Ree unital* $R(q)$, invariant under the Ree group. However, already for the case $q = 3$, a complete classification is missing. In 1981, Brouwer [2] constructed for $q = 3$ more than 130 further nonisomorphic unitals, i.e. 2-$(28, 4, 1)$ designs. He observed that the 2-rank of the constructed unitals is at least 19. Here, the p-rank of a design is defined as the rank of the incidence matrix between points and blocks of the design over the finite field GF(p). In 1998, McGuire, Tonchev and Ward [4] proved that indeed the 2-rank of a unital on 28 points is between 19 and 27 and that the Ree unital is the unique 2-$(28, 4, 1)$ design of 2-rank 19. In the same year, Jaffe and Tonchev [3] showed that there is no unital on 28 points of 2-rank 20 and there are exactly 4 isomorphism classes of unitals of rank 21.

Here, we present the complete classification by computer of unitals of 2-rank 22, 23 and 24. There are 12 isomorphism classes of unitals of 2-rank 22, 78 isomorphism classes of unitals of 2-rank 23, and 298 isomorphism classes of unitals of 2-rank 24.

Keywords
Combinatorial designs, finite geometry, combinatorial enumeration

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