

Quasi-symmetric $2-(41, 9, 9)$ designs and doubly even self-dual codes of length 40

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The residual design of a quasi-symmetric $2-(41, 9, 9)$ design with block intersection numbers 1 and 3 is a quasi-symmetric $1-(40, 8, 9)$ design with block intersection numbers 0 and 2. The incidence matrix of the latter generates a binary doubly even code of length 40. Using the database of binary doubly even self-dual codes of length 40 classified by Betsumiya, Harada and Munemasa [1], we prove that a quasi-symmetric $1-(40, 8, 9)$ design with block intersection numbers 0 and 2 is not extendable to a quasi-symmetric $2-(41, 9, 9)$ design with block intersection numbers 1 and 3, provided that it has a fixed-point-free automorphism of order 5 and 2-rank 20. This may be considered as a first step to prove the nonexistence of a quasi-symmetric $2-(41, 9, 9)$ design with block intersection numbers 1 and 3, and an analogue of the previous work [2], [3] for quasi-symmetric $2-(37, 9, 8)$ designs with block intersection numbers 1 and 3.

Keywords

quasi-symmetric design, binary code

References

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