Applications of Computer Algebra – ACA 2021
Virtual. Online | July 23-27, 2021
Session on *“Computer Algebra for Geometry and Combinatorics’*

Parameters of Affine Hermitian Grassmann Codes

Fernando Piñero González[1], ***Doel Rivera Laboy***[2] [fernando.pinero1@upr.edu]

[1] Department of Mathematics, University of Puerto Rico in Ponce, Ponce PR.
[2] Department of Mathematics, Pontificial Catholic University of Puerto Rico, Ponce, PR.

Abstract

The Grassmannian is arguably one of the most widely studied objects in Algebraic Geometry. The Grassmannian is embedded into projective space via the Plücker embedding. One of the techniques most often employed is to make a linear code from the projective points. This code is known as the Grassmann code. Nogin [1] determined the parameters of the Grassmann code for general field size q. The Grassmannian also has special subvarieties known as polar Grassmannians. A polar Grassmannian is a subvariety of the Grassmannian defined-only by subspaces isotropic under a bilinear or sesquilinar form. In case this form is Hermitian, it is known as the polar Hermitian Grassmannian. Cardinali and Guizzi determined the parameters of the polar Hermitian Grassmann code for 2–dimensional spaces. In this work we consider Affine Hermitian Grassmann codes. These codes may be considered as the linear codes defined from one of the affine maps of the polar Hermitian Grassmannian. We also consider these as evaluation codes. We determine their length, dimension and minimum distance. Affine Hermitian Grassmann codes also improve on the minimum distance os Affine Grassmann codes for similar parameters.

1 Introduction

The Grassmannian $(\mathcal{G}_{\ell,m})$ is the collection of all vector spaces of dimension ℓ of a vector space V of length m. We take $V = \mathbb{F}_q^m$. This is a highly interesting and well studied geometry with a rich algebraic structure.

It is well known that the Grassmannian may be embedded into a projective space through the Plücker embedding. In order to study the properties of the Grassmannian we make use of a linear code.

2 Preliminaries

Let q be a prime power, we let $\mathbb{F}_q$ denote the finite field of q elements.

Definition 1. *If M is a square matrix, then the minor $M^{I,J}$ is the determinant of the submatrix of M obtained from the rows I and columns J.*

Definition 2. *A matrix M over $\mathbb{F}_{q^2}$ is hermitian if $M_{j,i} = M_{i,j}^q$. We denote the collection of $\ell \times \ell$ Hermitian matrices over $\mathbb{F}_{q^2}$ by $\mathbb{H}^\ell(\mathbb{F}_{q^2})$.*

2.1 Affine Hermitian Grassman Code

For $\ell \geq 1$ we denote $X = [X_{ij}]$ as an $\ell \times \ell$ matrix of indeterminates where X_{ij}.

Definition 3. *$\Delta(\ell)$ is the set of all minors of the matrix X. That is:*

$$\Delta(\ell) := \{det_{I,J}(X^{I,J}), I, J \subseteq [\ell], |I| = |J|\}$$

Definition 4. *We define $\mathcal{F}(\ell)$ as the subspace of $\mathbb{F}_{q^2}$–linear combinations of elements of $\Delta(\ell)$.*

$$\mathcal{F}(\ell) := \{ \sum_{I,J\subseteq[\ell],\#I=\#J} f_{I,J} det_{I,J}(X^{I,J}), f_{I,J} \in \mathbb{F}_{q^2}\}$$

Definition 5. *The evaluation map of $\mathbb{F}_{q^2}[X]$ is the map*

$$Ev\text{: } \mathbb{F}_{q^2}[X] \to \mathbb{F}_{q^2}^n \text{ defined by } Ev(f) := (f(P_1), ..., f(P_n)).$$

Lemma 1.

$$\dim C^{\mathbb{H}}(\ell) = \binom{2\ell}{\ell}.$$

Theorem 2. *Suppose that $\ell \geq 2$. Then the minimum distance of the code $C^{\mathbb{H}}(\ell)$ is $q^{\ell^2} - q^{\ell^2-1} - q^{\ell^2-3}$.*

In the next two sections of the paper we work out a proof by induction of this fact. Our proofs will use the elementary techniques of polynomial evaluation and bounding the number of zeroes with the degree of a polynomial to determine the minimum distance of $C^{\mathbb{H}}(\ell)$.

References
[1] D.Yu. Nogin, *Codes associated to Grassmannians*, in: R. Pellikaan, M. Perret, S.G. Vladut (Eds.), Arithmetic Geometry and Coding Theory (Luminy, 1993), Walter de Gruyter, Berlin/New York, 1996, pp. 145–154.
[2] I. Cardinali and L. Giuzzi, Line Hermitian Grassmann codes and their parameters *Finite Fields and their Applications* **51** May 2018, 407–432.
[3] P. Beelen, S. Ghodepade and T. Høholdt,Affine Grassmann codes *IEEE Trans. on Info. Theory* **56**(7), 3166–3176. (2010)