Applications of Computer Algebra – ACA 2021
Virtual. Online | July 23-27, 2021
Session on *"Computer Algebra for Geometry and Combinatorics"*

Counting Arcs in the Projective Plane

Nathan Kaplan[1] [nckaplan@math.uci.edu]

[1] Department of Mathematics, University of California, Irvine, CA, USA

How many collections of n points in the projective plane over a finite field of size q have no 3 on a line? For $n \leq 6$, the formula is a polynomial in q [1]. For $7 \leq n \leq 9$, the formula is quasipolynomial [1,2,3]. For example, when $n = 7$ there is one polynomial formula that holds for odd q and a different one for even q. In this talk we will discuss the case $n = 10$ where the formula involves the number of $\mathbb{F}_q$-points on certain elliptic curves and K3 surfaces and is no longer quasipolynomial. We will emphasize computational and algorithmic aspects of this problem and will mention connections to coding theory. We will explain difficulties in adapting these ideas to deal with larger n. This is joint work with Isham, Weinreich, Lawrence, Kimport and Peilen.

Keywords
Arcs, projective planes, MDS codes.

References

[1] D. Glynn, *Rings of geometries. II.* J. Combin. Theory Ser. A 49 (1988), no. 1, 26-66.
[2] A. Iampolskaia; A. N. Skorobogatov; E. Sorokin, *Formula for the number of* $[9,3]$ *MDS codes*. IEEE Trans. Inform. Theory 41 (1995), no. 6, 1667-1671.
[3] N. Kaplan; S. Kimport; R. Lawrence; L. Peilen; M. Weinreich, *Counting arcs in the projective plane via Glynn's algorithm*. J. Geom. 108 (2017), no. 3, 1013-1029.