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Classifying Cubic Surfaces and Associated Objects

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Classification means determining the possible isomorphism types of a class of objects under the action of the symmetry group of the space in which the objects are considered. Classification is different from separation by invariants, which is weaker, as nonisomorphic objects may have the same set of invariants. One example is the problem of classifying cubic surfaces in three dimensional projective space over some field $\mathbb{F}$. The goal of classification is to provide all possible examples that can exist. This is helpful in the study of the properties of the objects, and it can provide insight into how the objects arise and how they are related. Classification is often hard, and computers can be used to support the work. This is especially true if the field $\mathbb{F}$ is assumed to be finite.

The talk will survey ongoing work with **Alice Hui**, **Nathan Kaplan** and **Fatma Karaoglu** regarding the classification of cubic surfaces and their associated structures such as quartic curves, Schläfli double sixes, general sets of points in a plane called arcs. Computer algebra comes in as an application of group theory to the problem of computing orbits. The groups are matrix groups over finite fields. The orbit algorithms are often based on partially ordered sets, in order to build the structures step-by-step.

What does this approach give? We can produce specific new examples of objects. We can produce computer-free theorems about the objects, for instance their symmety groups. This helps deepen previous knowledge that provided coarser information about the objects that was not at the level of classification up to isomorphism but separation by various geometric invariants such as the number of Eckardt points or the number of lines on the surface. Work over finite fields can be meaningful for algebraic objects in characteristic zero, as the example of the Eckardt surface shows, which was recently rediscovered over finite fields of odd characteristic.

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Cubic Surface, Projective Geometry, Quartic Curve, Arc, Computer Algebra